

TOPIC 2: Ratio and Proportion

A **ratio** is a comparison of two numbers by division. We usually write a ratio as a fraction because a fraction can be interpreted as division: $\frac{a}{b} = a \div b$, where b is not equal to 0.

Example 1: The average speed of a car is a ratio. Suppose you travel 120 miles in 2 hours. The average speed of the car is $\frac{120 \text{ miles}}{2 \text{ hours}}$, or 60 miles per hour.

Example 2: Data can be expressed as a ratio. Four out of five people surveyed prefer ice cream to Popsicles. This means that four-fifths $\left(\frac{4}{5}\right)$ of the people surveyed prefer ice cream to Popsicles.

There are other ways to write a ratio. For example, we can write 4 out of 5, 4 to 5, or 4:5 to indicate the ratio $\frac{4}{5}$.

A **proportion** is a statement of equality between two ratios. For example, the equation $\frac{2}{3} = \frac{8}{12}$ is a proportion.

For exercises 1-3, write the ratio in three other ways.

1. $\frac{3}{8}$ 2. 2:6 3. 12 to 15

For exercises 4-7, write the ratio that represents the statement in fraction form.

4. Andre Agassi won 4 out of 7 games at the French Open.
5. Jorge traveled 13 miles in 20 minutes.
6. Two out of 3 dentists recommend the use of SmileySmile toothpaste.
7. Patty answered 18 out of 25 questions correctly on her first quiz.

For exercises 8-11, reduce the fraction to create a proportion.

8. $\frac{4}{6} = -$ 9. $\frac{21}{28} = -$ 10. $\frac{15}{75} = -$ 11. $\frac{90}{120} = -$

We can solve proportions by applying the following property.

Fundamental Property of Proportions

For any real numbers a , b , c , and d , with $c \neq 0$ and $d \neq 0$,

$$\frac{a}{c} = \frac{b}{d} \text{ if and only if } ad = bc.$$

The products ad and bc are called **cross products** and are the result of cross multiplying.

Example 3: Solve $\frac{x}{20} = \frac{3}{4}$.

$$\frac{x}{20} = \frac{3}{4}$$

$$4x = 20 \cdot 3 \quad \text{[cross multiplying]}$$

$$4x = 60$$

$$\frac{4x}{4} = \frac{60}{4} \quad \text{[dividing both sides by 4]}$$

$$x = 15$$

Check the answer: $\frac{x}{20} = \frac{3}{4}$

$$\frac{15}{20} = \frac{3}{4} \quad \text{[letting } x = 15\text{]}$$

$$15 \cdot 4 = 20 \cdot 3$$

$$60 = 60 \quad \text{[it checks]}$$

For exercises 12-15, solve the proportion. Show a check of your answer.

12. $\frac{12}{x} = \frac{240}{300}$

13. $\frac{9}{12} = \frac{x}{18}$

14. $\frac{x-3}{5} = \frac{x+2}{4}$

15. $\frac{7}{x-2} = \frac{4}{x+3}$

Word problems sometimes can be solved using proportions. When using a proportion to solve a problem, it is important that you set up your ratios similarly. The answer to a word problem should be written in a complete sentence.

Example 4: The ratio of male students to female students at a college is 5 to 4. If there is a total of 360 male students, how many female students are at the college?

Example 4: Here it will be important that we set up the ratios the same way. We will
 (continued) set them up as $\frac{\text{male}}{\text{female}}$. So the proportion is $\frac{5}{4} = \frac{360}{f}$, where f represents
 the number of female students at the college

$$\begin{aligned}\frac{5}{4} &= \frac{360}{f} \\ 5f &= 4 \cdot 360 \\ 5f &= 1440 \\ f &= 288\end{aligned}$$

There are 288 female students at the college.

Example 5: A sum of \$1250 is divided between two people in the ratio of 2 to 3.
 How much did each person receive?

We let x be the amount received by one person. Therefore the other person received $\$1250 - x$. Because the money is divided between the two people in the ratio of 2 to 3, we have the following proportion:

$$\begin{aligned}\frac{x}{1250 - x} &= \frac{2}{3} \\ 3x &= 2(1250 - x) \\ 3x &= 2500 - 2x \\ 5x &= 2500 \\ x &= 500\end{aligned}$$

One person received \$500. The other person received $\$1250 - \$500 = \$750$.

For exercises 16-20, solve the word problem using a proportion. Show all your work and write your answer in a complete sentence.

16. Jamie used 10 gallons of gasoline to travel 170 miles. How much gasoline will she need to travel 238 miles?
17. It was reported that a flu epidemic is affecting 6 out of every 10 college students in a certain part of the country. At this rate, how many college students will be affected at a university of 15,000 students?
18. A pre-election poll indicated that three out of every seven eligible voters were going to vote in an upcoming election. At this rate, how many people are expected to vote in a city of two hundred ten thousand eligible voters?

19. A certain medicine is prescribed in an amount proportional to a patient's body weight. If a patient weighing 70 kilograms requires 210 milligrams of medicine, how much medicine is required for a patient weighing 80 kilograms?
20. At a certain high school, there are 3 females athletes for every 5 males athletes. If there are 2160 athletes at the school, how many female athletes are there?

The Number p

One of the most famous ratios in mathematics is the number p . In this exploration you will see how close you can approximate p by taking measurements of circles found in the room.

Copy and complete the table below in your notebook. Begin by selecting five circular objects in the room. You will not be able to measure the diameter of a spherical object accurately so do not choose spherical objects. Now record the name of each object in the first column. To measure the circumference of the object, take a piece of string and place it around the edge of the circle, marking off the length of the circumference on the string. Then place the string along side a ruler to measure the circumference of the circle. As closely as you can, determine the center of the circle and then measure the diameter, using the string if necessary. Be sure to measure the circumference and diameter using the same units. Divide the circumference by the diameter and record that number, accurate to five decimal places, in the last column. Compare your results with other students.

	Object	Circumference (C)	Diameter (d)	$\frac{C}{d}$
1				
2				
3				
4				
5				

21. A formula for the circumference of a circle is $C = pd$. Solve the formula for p . Define p using the word *ratio*.
22. The values in the last column of the table are approximations of p . How could you have gotten better approximations?
23. The number p is approximately 3.14. How close to p are the values in the last column? How close to p is the average of the values in the last column?