

TOPIC 4: Exponents and Scientific Notation

We use exponents to write repeated multiplication. For example, $2^3 = 2 \cdot 2 \cdot 2$. In general, for any positive integer n ,

$$a^n = a \cdot a \cdot a \cdots a \quad \text{where there are } n \text{ factors of } a.$$

In the expression a^n , which is read “ a to the n th power,” a is the **base** and n is the **exponent**. We can also extend this definition to exponents that are not positive integers. For $a \neq 0$, we have

$$a^0 = 1 \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

Note that 0^0 is undefined.

We can simplify expressions that contain exponents by using the following properties.

Properties of Exponents

Assume that all expressions are defined.

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(ab)^n = a^n b^n$$

For exercises 1-10 use the properties of exponents to simplify each expression. Write your answers with positive exponents only. Assume that all expressions are defined.

1. $x^5 x^3$

2. $\frac{7^3}{7^2}$

3. $(5^2)^3$

4. $(-2x)^3$

5. $(x^3 y^2)^3 (3xy^7)^0$

6. $\left(\frac{6a^2 b}{2ab^3}\right)^3$

7. $(3x)^{-2}$

8. $\left(\frac{2}{x^{-3}}\right)^{-1}$

9. $(2^4 \cdot 3^8)(2^2 \cdot 3^{-4})$

10. $(a^{-5} b^7 c^{-2})(a^{-3} b^{-2} c^6)^2$

11. Suppose $x = 2$, $y = 3$, and $w = -2$. Find the value of $\left[\frac{(xy)^2 w^{-3}}{(x^{-3} y^5 w^{-2})^{-2}}\right]^{x+w}$.

12. For what values of a and n will a^n represent a negative number?

13. For what values of a and n will a^{-n} represent a negative number?

14. Does $ab^n = (ab)^n$? Explain.
15. Does $-a^n = (-a)^n$? Explain.
- 16a. Find values for a and b such that $(a+b)^2 = a^2 + b^2$.
- b. Does $(a+b)^2 = a^2 + b^2$? Explain.
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Scientific notation uses of the properties of exponents to simplify the writing of very large or very small numbers. In *scientific notation*, a number is written as

$$N \times 10^n \text{ where } 1 \leq N < 10 \text{ and } n \text{ is an integer.}$$

Example 1: Write 425,000 using scientific notation.

Note that $10^5 = 100,000$ and that $4.25 \times 100,000 = 425,000$. In scientific notation, 425,000 is written as 4.25×10^5 .

For exercises 17-20, write the number using scientific notation.

17. 38,500 18. 0.00000472 19. 101 20. 0.23

For exercises 21-24, write the number without using scientific notation.

21. 2.3×10^{-4} 22. 1.45×10^6 23. 4×10^{-1} 24. 3.03×10^4

For exercises 25-28, multiply or divide. Write your answers in scientific notation.

25. $(4 \times 10^7)(3.1 \times 10^5)$ 26. $\frac{4.2 \times 10^{-2}}{3 \times 10^5}$
27. $\frac{1}{20000}$ 28. $(7 \times 10^{17})^2$