TOPIC 5: Compound Interest

Suppose you place $1000 in a savings account that earns 10% interest per year. After one year you will have earned $100 in interest and will now have a total of $1100 in the account. If you decide to keep the money in the account for another year, you will earn 10% interest on $1100. After this second year you will have earned an additional $110 in interest and your account will now have $1210. Interest of this type is called ***annual compound interest*** because every year the interest in compounded, or added to the amount in the account at the end of the previous year.

Let's look at this scenario in an algebraic way, using a table of values and equations. In the algebra column of the table, *A* represents the amount in the account, *P* represents the principal (the amount of money originally invested), and *r* represents the annual interest rate.

Time Period	Algebra	Arithmetic
After 1 year	$A = P + P \cdot r$ $A = P(1+r)$	$A = 1000 + 1000(.10)$ $A = 1000(1+.10)$ $A = 1000(1.10)$ $A = 1100$
After 2 years	$A = P(1+r) + P(1+r) \cdot r$ $A = P(1+r)(1+r)$ $A = P(1+r)^2$	$A = 1000(1.10) + 1000(1.10)(.10)$ $A = 1000(1.10)(1+.10)$ $A = 1000(1.10)(1.10)$ $A = 1000(1.10)^2$ $A = 1210$
After 3 years		
After 4 years		

1. Copy the table above into your notebook. Fill in the algebra and arithmetic columns for 3 and 4 years.

2. Write a formula for the amount *A* in the account after *t* years

---

Some accounts are compounded more than once a year. Accounts can be compounded semiannually, quarterly, monthly, daily, and so on. The formula for the amount *A* in an account after *t* years with principal *P* earning an annual interest rate *r* compounded *n* times per year is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

3. Let's invest a million dollars for one year at an annual interest rate of 10% and see how the number of times we compound the interest affects our account. Copy and complete the table below in your notebook. Write the amounts to the nearest dollar.

Type of compounding, n times per year	$A = P\left(1+\frac{r}{n}\right)^{nt}$
Annually $n =$	
Semi-annually $n =$	
Quarterly $n =$	
Monthly $n =$	
Daily (365 days) $n =$	
Hourly $n =$	
Each minute $n =$	
Each second $n =$	

4. What conclusions can you draw about more frequent compounding of interest?

5. Does there seem to be a limit on how much you can earn by increasing the number of times you compound per year? Explain.

A Checkerboard Problem

You will use an 8 x 8 grid provided by your instructor as your checkerboard. Imagine putting a penny on the top left square. Represent this by writing the number 1 in the square. What power of two equals one? Now move to the next square to the right. Imagine placing two pennies on this square. Write in the square the power of two that represents two pennies. Continue doubling the number of pennies as you move to the next square and write in each square the power of two that represents the number of pennies on that square.

6. How much money is on the square with 2^8 pennies?

7. In which row and column of the checkerboard is the square with $40.96?

8. Which square will be the first to exceed one million dollars?

9. How many dollars are there on the square with 2^{63} pennies?

10. What is the total amount of money on the checkerboard?