TOPIC 6: Solving Linear Equations

An ***equation*** is a mathematical sentence stating that two numbers or expressions are equal. Every equation has two sides, one on either side of the equal sign. We can solve equations by applying the following properties.

> Properties of Equality
> Let a, b, and c be real numbers. Then $a + c = b + c$ if and only if $a = b$.
>
> Let a, b, and c be real numbers with c ? 0. Then $a \cdot c = b \cdot c$ if and only if $a = b$.

These properties allow us to perform the same operation (addition or multiplication) to both sides of an equation. The properties of equality also hold for the operations of subtraction and division. Our goal in solving equations is to use these properties to isolate the variable on one side.

A ***linear equation in one variable*** is an equation that contains a variable to the first power and no terms of higher degree (terms that have an exponent greater than one). For example, the equation $5 - x = 7$ is a linear equation.

Example 1: Solve $5 - x = 7$.

$$5 - x = 7$$
$$5 - x - 5 = 7 - 5 \qquad \text{[subtracting 5 from both sides]}$$
$$-x = 2$$
$$(-1) \cdot (-x) = (-1) \cdot (2) \qquad \text{[multiplying both sides by –1]}$$
$$x = -2$$

Check the answer:

$$5 - x = 7$$
$$5 - (-2) = 7 \qquad \text{[letting x = –2]}$$
$$5 + 2 = 7$$
$$7 = 7 \qquad \text{[it checks]}$$

For exercises 1-12, solve the equation. Show a check of your answer.

1. $3x - 1 = 5$
2. $2x + 3 = 5x$
3. $4 - 7x = -10$
4. $\frac{x}{2} = \frac{3}{4}$
5. $\frac{5}{4} = \frac{5}{2x}$
6. $0.2x - 0.6 = -1.2$
7. $\frac{1}{2}x - \frac{3}{4} = \frac{5}{4}$
8. $2(x - 1) = 3(x + 2)$
9. $x(x - 2) = x^2 - 3(x + 5)$
10. $\frac{1}{3}x - (x + 2) = 2\left(\frac{1}{3}x + 2\right)$

11. $-\frac{x}{5}+1=2x-4(x+1)$

12. $x(3x-2)-2(x+4)=4(x^2-3x)+x(3-x)$

For exercises 13 and 14, solve the equation using *cross products.*

13. $\frac{x}{10}=\frac{x+1}{25}$

14. $\frac{1}{x-1}=\frac{3}{1-x}$

15. Can you solve the equation $6-\frac{1}{x}=\frac{3}{4}$ using *cross products*? If you can, solve it; if you can't, explain why not, and solve the equation by a different method.

Example 2: Jeremy receives an allowance of $12.50 per week, plus $1.75 each time he does the dishes. If he earned $35.25 last week, how many times did he do the dishes?

We let *d* represent the number of times Jeremy did the dishes last week and use the given information to write the following equation:

Earnings last week = 12.50 + 1.75(Number of times he did the dishes)
35.25 = 12.50 + 1.75*d*

We can now solve the equation.

$$35.25 = 12.50 + 1.75d$$
$$35.25 - 12.50 = 12.50 + 1.75d - 12.50 \quad \text{[subtracting 12.50 from both side]}$$
$$22.75 = 1.75d$$
$$\frac{22.75}{1.75} = \frac{1.75d}{1.75} \quad \text{[dividing both sides by 1.75]}$$
$$13 = d$$

Jeremy did the dishes 13 times last week.

For exercises 16-19, solve the word problem using an equation. Write your answer in a complete sentence.

16. Darius went to an art store with $20 and bought two sketch pads, which were $3.50 each. He also wanted to get some colored pencils which cost $1.50 each. What is the greatest number of pencils he could buy (ignore sales tax)?

17. Jessica earns $8 an hour regularly and $12 an hour for overtime (anything over 40 hours in a week). How much would she make if she worked 42 hours in a week? How many hours would she have to work to make $500 in a week?

18. Gregory has four tests in his math class. If he received scores of 75, 85, and 90 on the first three, what score would he need on the fourth test to have an average test score of 85 in the class?

19. A man had three equal piles of apples. He discovered 8 rotten apples, threw them away, and divided the remaining apples into two equal piles. If the final two piles contained 17 apples each, how many apples were in each of the original three piles?

Many applications of mathematics involve relationships among two or more quantities. An equation that represents such a relationship is known as a ***formula***. For example, the formula $C = 2pr$ gives the circumference C of a circle with radius r. We can solve a formula for a specified variable in the same way we solve equations.

Example 3: The formula $A = lw$ gives the area A of a rectangle with length l and width w. Solve the formula for w.

When we solve for w, we can consider the other quantities as constants.

$$A = lw$$

$$\frac{A}{l} = \frac{lw}{l} \qquad \text{[dividing both side by } l\text{]}$$

$$\frac{A}{l} = w$$

Example 4: The formula $S = 2pr^2 + 2prh$ gives the surface area S of a cylinder with base radius r and height h. Solve the formula for h.

We consider all quantities except h as constants and solve the formula for h using the properties of equality.

$$S = 2pr^2 + 2prh$$

$$S - 2pr^2 = 2pr^2 + 2prh - 2pr^2 \qquad \text{[subtracting } 2pr^2 \text{ from both sides]}$$

$$S - 2pr^2 = 2prh$$

$$\frac{S - 2pr^2}{2pr} = \frac{2prh}{2pr} \qquad \text{[dividing both sides by } 2pr\text{]}$$

$$\frac{S - 2pr^2}{2pr} = h$$

For exercises 20-27, solve the formula for the indicated variable.

20. $d = rt$, for t

21. $P = 2l + 2w$, for w

22. $y = 3x + 1$, for x

23. $2m - 3n = 5$, for n

24. $Ax + By = C$, for y

25. $p + s(p + q) = 2s$, for p

26. $a = b(a + c)$, for a

27. $\frac{1}{t} = \frac{1}{r} + \frac{1}{s}$, for r