TOPIC 7: Solving Inequalities

An ***inequality*** is a mathematical sentence stating that one number or expression is less than, or greater than, another. Every inequality includes a symbol of inequality. If a and b are real numbers, then

$a < b$ means *a is less than b,*
$a = b$ means *a is either less than or equal to b,*
$a > b$ means *a is greater than b,*
$a = b$ means *a is either greater than or equal to b.*

If a is less than b, we can also say that b is greater than a. The other inequalities have similar interpretations.

We call the symbols $<$ and $>$ *strict inequalities* because they imply that one side of the inequality must be less than the other side, whereas the symbols $\leq$ and $\geq$ allow for the two sides to possibly be equal.

The ***solution set*** of an inequality is the set of real numbers that makes the inequality true. Solving inequalities is very similar to solving equations. We can use the following properties to solve inequalities.

<u>Properties of Inequalities</u>
Let a, b, and c be real numbers. Then
$a + c < b + c$ if and only if $a < b$
$a + c > b + c$ if and only if $a > b$
If c is positive, then
$ac < bc$ if and only if $a < b$
$ac > bc$ if and only if $a > b$
If c is negative, then
$ac > bc$ if and only if $a < b$ (Note the direction switch of the inequality)
$ac < bc$ if and only if $a > b$ (Note the direction switch of the inequality)

The properties of inequality also hold for $\leq$ and $\geq$.

Remember: when multiplying or dividing by a negative number in an inequality you must switch the direction of the inequality.

Example 1: Solve the inequality $-2x + 4 < -6$.

We use the properties of inequality:

$$-2x + 4 < -6$$
$$-2x < -10$$
$$\frac{-2x}{-2} > \frac{-10}{-2}$$ [dividing both sides by –2, switching the direction of the inequality]
$$x > 5$$

Example 1: (continued) The solution of the inequality is the set of all real numbers greater than 5. We can write the solution set in ***set-builder notation*** as $\{x \mid x > 5\}$, which is read, "the set of all real numbers *x* such that *x* is greater than 5."

For exercises 1-4, solve the inequality and write the solution set in set-builder notation.

1. $4x - 3 \leq 21$
2. $-3x + 1 > -2$
3. $5x - 4 > 2 + 3x$
4. $\frac{x-1}{2} + \frac{x+3}{5} \geq \frac{1}{10}$

We often use ***interval notation*** to write the solution set of an inequality. The interval notation for several inequalities is shown below. We assume *a* and *b* are real numbers.

Inequality	Interval
$x < a$	$(-\infty, a)$
$x \leq a$	$(-\infty, a]$
$x > a$	(a, ∞)
$x \geq a$	$[a, \infty)$

The set of real numbers **R** is represented by the interval $(-\infty, \infty)$.

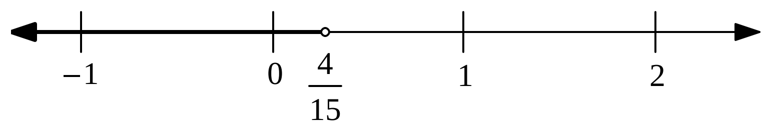
Example 2: Solve the inequality $\frac{3}{4}x < \frac{1}{5}$. Write the solution set in interval notation and graph the solution set on a number line.

Using the properties of inequalities we have:

$$\frac{3}{4}x < \frac{1}{5}$$
$$\left(\frac{4}{3}\right)\frac{3}{4}x < \frac{1}{5}\left(\frac{4}{3}\right)$$
$$x < \frac{4}{15}$$

We can write the solution set in interval notation as $\left(-\infty, \frac{4}{15}\right)$. To graph the solution set on a number line, we place an open circle on $\frac{4}{15}$ and darken the number line to the left of the open circle (see figure below).

Example 2:
(continued)



The open circle indicates that $\frac{4}{15}$ is not part of the solution set. An open circle occurs when we have a strict inequality; we use a closed circle if an inequality contains the symbols = or =.

For exercises 5-8, solve the inequality. Write the solution set in interval notation and graph it on a number line.

5. $2x - 9 > x + 4$

6. $\frac{3}{4}x - \frac{5}{6}x \le \frac{3}{8}$

7. $4(x-5) - 2(x-1) < 14$

8. $5x + 8x \ge 3x - 10$

For exercises 9-13, solve the word problem using an inequality. Write your answer in a complete sentence.

9. Five more than three times a number is less than 26. Find the numbers that satisfy this relationship.

10. Suppose that the perimeter of a rectangle is to be no greater than 70 inches and the length of the rectangle must be 20 inches. Find the possible widths of the rectangle.

11. Sue bowled 132 and 160 in her first two games. What must she bowl in the third game to have an average of at least 150 for the three games?

12. This semester Micah has scores of 87, 81, and 74 on his first three algebra tests. What score must he get on the fourth test to have an average of 85 or higher for the four tests?

13. Mona has $1000 to invest. If she invests $500 at an annual interest rate of 8%, at what annual rate must she invest the other $500 so that the two investments together yield more than $100 of yearly interest?