

TOPIC 8: Word Problems

Word problems are often difficult to solve. You can use the following five-step method to help you set up and solve word problems.

Step 1: *Familiarize* yourself with the problem. Read the problem carefully. List the information given and identify the quantity (or quantities) to be found. Draw any diagrams or figures if needed.

Step 2: *Translate* the information into mathematical language. Use a variable(s) to label the quantity (or quantities) to be found. Use the given information to write an equation or inequality.

Step 3: *Solve* the equation or inequality.

Step 4: *Check* your answer(s). Does the answer satisfy the conditions of the problem? Does the answer make sense?

Step 5: *State your answer* clearly, using a complete sentence.

Example 1: Last week, Bill sold stock worth \$35 per share. This was 17 dollars less than twice what he had originally paid for it. What price did he pay for each share of stock?

Step 1: *Familiarize.* Bill sold the stock for \$35 per share. We must find the price per share he paid for the stock.

Step 2: *Translate.* Since we must find the price per share Bill paid for the stock, we let p represent the price per share he paid. Bill sold the stock for \$35 per share and this was \$17 less than twice what he had originally paid for it. We can use these conditions to write an equation:

$$\begin{aligned}\text{Selling price per share} &= \text{Twice the original price per share} - 17 \\ 35 &= 2p - 17\end{aligned}$$

Step 3: *Solve.*

$$\begin{aligned}35 &= 2p - 17 \\ 52 &= 2p \\ 26 &= p\end{aligned}$$

Step 4: *Check.* If Bill paid \$26 per share for the stock, then twice that is \$52. Seventeen less than that is \$35, so the answer checks.

Step 5: *State your answer.* Bill paid \$26 per share for the stock.

For exercises 1-8, solve the word problem using the five-step method. Identify the steps as you work.

1. In a class of 50 students, the number of women is 2 more than 5 times the number of men. How many women are in the class?
 2. Jenna is running in a 10-km race. She is twice as far from the finish line as she is from the start. How far has she run?
 3. Doug paid \$72 for a portable CD player at a 20%-off sale. What was the regular price of the portable CD player?
 4. Mack's Parking Garage charges \$4.00 for the first hour and \$2.50 for each additional hour. For how long has a car been parked when the charge exceeds \$16.50?
 5. A soccer team won 10 more games than it lost, and the ratio of its wins to its losses was 7 to 5. How many games did the team win? How many games did it lose?
 6. A motel rents double rooms for \$42 a day and single rooms for \$22 a day. If a total of 55 rooms were rented for \$2010, how many of each kind were rented?
 7. Four cheeseburgers and five milkshakes cost a total of \$25.50. Two milkshakes cost \$1.75 more than one cheeseburger. What is the cost of a cheeseburger? What is the cost of a milkshake?
 8. For the first three days of a four-day golf tournament, Vera shot rounds of 73, 75, and 71. What must she shoot on the fourth day to average 72 or less for the tournament?
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As we see in the following example, sometimes we can use a formula to solve word problems.

Example 2: Two airplanes leave San Francisco at the same time, flying in opposite directions. If one travels at 500 mph and the other travels at 600 mph, how long will it take for them to be 1925 miles apart?

Step 1: *Familiarize.* The speeds of the planes are given. We must find the number of hours they will fly to be 1925 miles apart.

Step 2: *Translate.* We can use the familiar formula $d = r \cdot t$, where d is the distance traveled, r is the rate (speed), and t is the time traveled. We let t represent the number of hours it takes for the planes to be 1925 miles apart. Since each plane travels for t hours, the distance traveled by the first plane is $500t$ and the distance traveled by the second plane is $600t$. We organize this information in the table below.

<i>Example 2:</i> <i>(continued)</i>		Distance	Rate (speed)	Time
	Plane 1	$500t$	500	t
	Plane 2	$600t$	600	t

The distance between the planes is just the distance traveled by the first plane plus the distance traveled by the second plane because the planes are flying in *opposite* directions. (Make a diagram!) We can now write an equation:

$$\begin{aligned} \text{Distance between planes} &= \text{Distance of plane 1} + \text{Distance of plane 2} \\ 1925 &= 500t + 600t \end{aligned}$$

Step 3: Solve.

$$\begin{aligned} 1925 &= 500t + 600t \\ 1925 &= 1100t \\ 1.75 &= t \end{aligned}$$

Step 4: Check. If the first plane travels for 1.75 hours (1 hour and 45 minutes), then it will travel a distance of $500(1.75) = 875$ miles. If the second plane travels for 1.75 hours, then it will travel a distance of $600(1.75) = 1050$ miles. So the distance between the planes will be $875 + 1050 = 1925$ miles.

Step 5: State your answer. The planes will travel for 1 hour and 45 minutes.

For exercises 9-20, solve the word problem using the five-step method. Identify the steps as you work. You may need a formula to solve some of the problems.

- Two cars leave Santa Fe traveling in opposite directions. One car is traveling 4 mph faster than the other. If they are 520 miles apart after 5 hours, what is the speed of each car?
- The perimeter of a rectangular swimming pool is 120 feet. If the length of the pool is three times the width, what are the dimensions of the pool?
- The height of a triangle is 15 cm. For what lengths of the base is the area of the triangle at most 60 cm^2 ?
- Sandy left a town traveling in her car at 45 mph. One hour later, Monica leaves the same town in her car traveling the same route at 50 mph. How long will it take Monica to overtake Sandy?

13. A movie theatre had a certain number of tickets to give away. Five people received tickets. The first person got one-third of the tickets. The second person got one-fourth of the tickets. The third person got one-fifth of the tickets. The fourth person got eight tickets, and there were five tickets left over for the fifth person. How many tickets were given away?
14. Omar invested a certain amount of money in an account paying 10% interest per year and \$250 more than that amount in an account paying 11% interest per year. If he earns a total yearly interest of \$153.50 from the two investments, how much did he invest in each account?
15. The width of a rectangle is 17 inches. What lengths will make the perimeter greater than 86 inches?
16. The Jacksons are planning to sell their home. They must pay the realtor a 6% commission on the sale price. They must also pay the escrow company \$5000 in closing costs. If the Jacksons want to make a profit of \$200,000 from the sale, for how much should they sell their home?
17. Jane is 5 years older than Sue. In eight years, Jane will be four years younger than twice Sue's current age. What are Jane and Sue's ages?
18. The perimeter of a triangle is 240 mm and the lengths of the sides are consecutive even integers. What are the lengths of the sides?
19. The tens digit of a 2-digit number is two less than the units digit. If the digits are reversed, the sum of the new number (the one with the digits reversed) and the original number is 154. What is the original number?
20. An on-line company sells CD's. The company charges \$8.49 for the first CD ordered and \$3.99 for all others. For shipping and handling, the company charges \$2.47 for the first CD, \$2.28 for the second CD, and \$1.99 for all others. The total cost of a shipment (excluding taxes) was \$65.07. How many CD's were in the shipment?