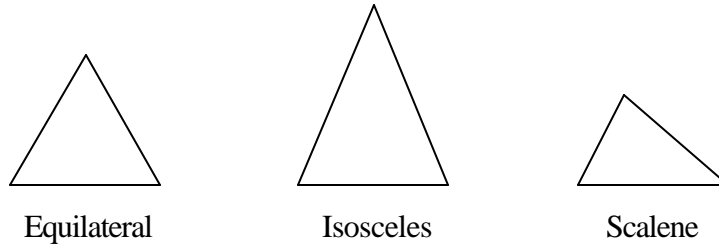


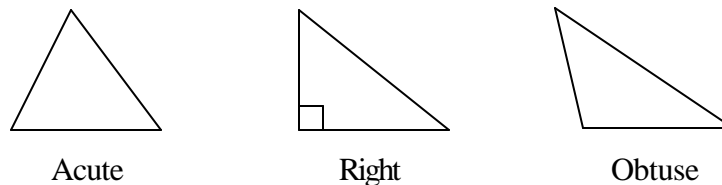
TOPIC 10: Triangles

There are several ways to classify triangles. One way uses the lengths of the sides: an **equilateral** triangle has three equal sides; an **isosceles** triangle has at least two equal sides; and a **scalene** triangle has three sides of different lengths.



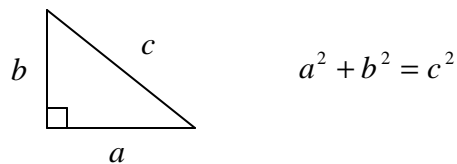
An important theorem from geometry states: *If two sides of a triangle are congruent (have the same length), then the angles opposite those sides are congruent (have the same measure).*

We can also classify triangles by the measures of the angles: an **acute** triangle contains three angles each less than 90° ; a **right** triangle contains a 90° angle; and an **obtuse** triangle contains an angle greater than 90° .



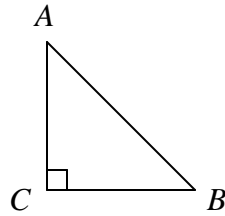
Recall that the sum of the measures of the angles in a triangle is 180° .

One of the most famous theorems in geometry is the *Pythagorean Theorem*. The theorem states: *In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.* See the figure below:

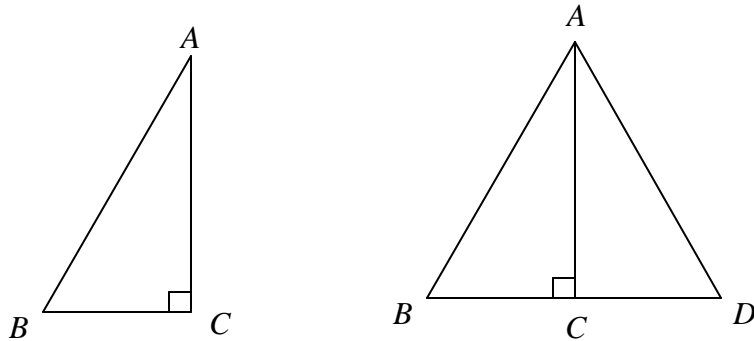


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1. What are the measures of the angles in an equilateral triangle? Why?
 2. Is an equilateral triangle an isosceles triangle? Explain.
 3. Is an isosceles triangle an equilateral triangle? Explain.
 4. In a right triangle, the two acute angles are complementary. Why?

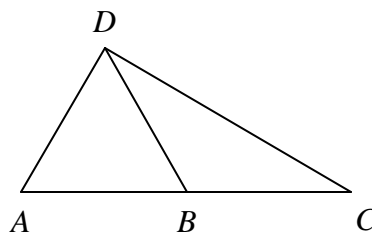
5. In $\triangle ABC$ shown below, suppose $AC = a$ and $BC = a$.



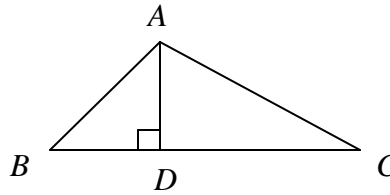
- Find $m\angle A$ and $m\angle B$.
 - Find AB .
6. If the hypotenuse of an isosceles right triangle has length 12 in, find the lengths of the legs.
7. In $\triangle ABC$ shown below on the left, $m\angle B = 60^\circ$ and $BC = a$. Now imagine making an exact copy of the triangle and attaching it to the original triangle as show below on the right:



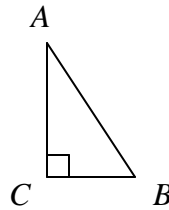
- In $\triangle ABC$ shown above on the left, find $m\angle A$.
 - What type of triangle is $\triangle ABD$? Explain.
 - Find AB .
 - Find AC .
8. If the hypotenuse of a 30° - 60° - 90° triangle has length 24 cm, find the lengths of the legs.
9. In $\triangle ACD$ shown below, $\triangle ABD$ is equilateral and $BD = BC$. Find $m\angle BDC$.



10. In $\triangle ABC$ shown below, $AD = BD$ and $m\angle BAC = 105^\circ$. Find $m\angle CAD$.

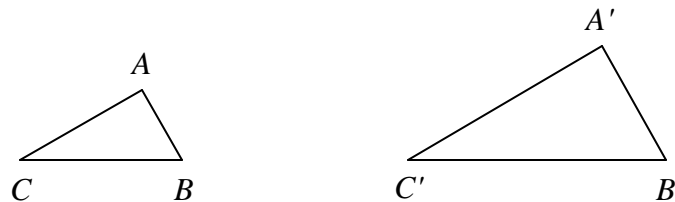


11. In $\triangle ABC$ shown below, suppose $AB = 18$ cm and $AC = 15$ cm. Find BC .



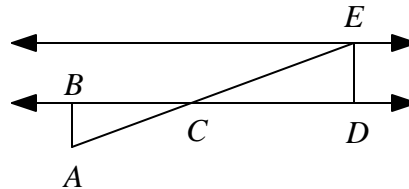
12. A baseball infield is a square with 90 ft between the bases.
- How far is it from home plate to second base?
 - How far is it from a point exactly midway between second and third base to first base?
13. The Gateway Arch in St. Louis is approximately 670 ft high. If you are 500 ft away from the point on the ground directly below the top of the arch, how far are you away from the top of the arch?
14. A house has a flat roof. A TV antenna on the roof is supported by a wire running from the top of the antenna to the roof. If the wire is 1 m longer than the height of the antenna and is anchored 3 m from the foot of the antenna, how tall is the antenna?

Two triangles are *similar* if they have the same shape. Although similar triangles need not be the same size, they must have congruent corresponding angles. Furthermore, the ratios of the lengths of corresponding sides are equal. We use the symbol “ \sim ” for “is similar to.” In the figure shown below, $\triangle ABC \sim \triangle A'B'C'$:

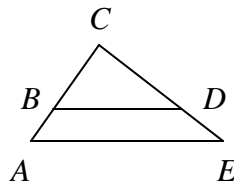


$$\angle A \cong \angle A', \angle B \cong \angle B', \angle C \cong \angle C' \text{ and } \frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

15. A person standing 6 ft tall casts a shadow of length 4 ft. Nearby, in direct line with the person, a tree casts a shadow of 24 feet. Use similar triangles to find the height of the tree.
16. The diagram below shows how surveyors laid out similar triangles along a ravine. The width of the ravine is DE .



- a. Explain why $\triangle ABC \sim \triangle EDC$.
- b. If $AB = 60$ ft, $BC = 160$ ft, and $CD = 220$ ft, find the width of the ravine.
17. In the figure shown below, \overline{BD} is parallel to \overline{AE} .



- a. Prove: $\angle CBD \cong \angle CAE$.
- b. Explain why $\triangle CBD \sim \triangle CAE$.
- c. If $AB = 2$ cm, $BC = 5$ cm, and $BD = 8$ cm, find AE .
18. Two triangles are **congruent** if they have the same size *and* shape. In congruent triangles, corresponding angles are congruent and corresponding sides are congruent.
- a. Are similar triangles congruent? Explain.
- b. Are congruent triangles similar? Explain. If yes, what is the ratio of corresponding sides?