TOPIC 12: More on Lines

We have seen that $4x + 2y = 12$ can be written as $y = -2x + 6$. This line has slope –2 and y-intercept (0, 6). In general, when we express the equation of a line in ***slope-intercept form***,

$$y = mx + b,$$

the slope m is the coefficient of x and the constant term b is the y-coordinate of the y-intercept $(0, b)$.

We can also write the equation of a line in another way. Suppose a line with slope m contains the point (x_1, y_1). If (x, y) is any other point on the line, then

$$m = \frac{y - y_1}{x - x_1}.$$

Multiplying both sides of the last equation by $x - x_1$ we obtain the ***point-slope form*** of the equation of the line:

$$y - y_1 = m(x - x_1).$$

Example 1: A line contains the points (–1, 2) and (3, 10). Find an equation of the line and find the y-intercept of the line.

We first find the slope of the line and then use the point-slope form. Note that

$$m = \frac{10 - 2}{3 - (-1)} = \frac{8}{4} = 2$$

Since the line contains the point (3, 10), we have

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 10 &= 2(x - 3) \\ y &= 2x + 4 \qquad \text{[solving for } y\text{]} \end{aligned}$$

We see that the last equation is in slope-intercept form, so the y-intercept of the line is the point (0, 4). We could have used the point (–1, 2) instead of the point (3, 10) and obtained the same result.

For exercises 1-3, solve for y and find the slope and the y-intercept of the line.

1. $4x = y + 7$ 2. $3x + 2y = 10$ 3. $\frac{x}{3} - 3y = 0$

For exercises 4-7, write the slope-intercept form of the equation of the line that satisfies the given conditions. Find the y-intercept of the line.

4. $m = -\frac{1}{2}$, contains the point (2, 4).
5. $m = 0$, contains the point (4, –7).
6. Contains the points (–3, 0) and (–1, 5).
7. Contains the points (–3, 2) and (0, –5).

8. The monthly cost of a regular checking account at a local bank is \$5.00 plus \$0.15 per check written. Let x represent the number of checks written and let y represent the monthly cost of the checking account.
   a. Write an equation that gives the monthly cost y to write x checks.
   b. How many checks were written if the cost last month was \$8.45?

9. Suppose the pressure 110 feet beneath the ocean's surface is 4 atmospheres (atm), and at a depth of 200 feet the pressure is 7 atm. Let x represent the depth beneath the ocean's surface and let y represent the pressure. Assume there is a linear relationship between depth and pressure.
   a. Write an equation that gives the pressure y at a depth of x feet.
   b. What is the pressure at a depth of 690 feet?

10. Water freezes at 0ºC, which is 32ºF, and it boils at 100ºC, which is 212ºF. Let x represent the temperature in degrees Celsius (ºC) and y represent the temperature in degrees Fahrenheit (ºF). Assume there is a linear relationship between x and y.
    a. Write an equation that gives y in terms of x.
    b. The average body temperature is 98.6ºF. What is the average body temperature in degrees Celsius?
    c. Is there a number z such that zºF = zºC?

Two linear equations considered together is a ***linear system***. For example, the pair of equations

$$x + y = 1$$
$$2x - y = -10$$

is a linear system in two variables. A ***solution*** of a linear system is an ordered pair (x, y) that makes *both* equations true. The ordered pair $(-3, 4)$ is a solution of the linear system above because it satisfies both equations:

$$\begin{aligned} x + y &= 1 \\ (-3) + 4 &= 1 \\ 1 &= 1 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x - y &= -10 \\ 2(-3) - 4 &= -10 \\ -10 &= -10 \end{aligned}$$

We can also have linear systems with more than two equations and more than two variables.

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11. Graph the equations in the linear system above on the same coordinate system. Can you determine the solution of the system from the graph? Explain.

12. Does the linear system above have more than one solution? Explain.

13. Graph the linear system $\begin{matrix} x + y = 3 \\ x + y = 7 \end{matrix}$. Does the system have a solution? Explain.

14. Graph the linear system $\begin{matrix} 2x - y = 4 \\ -6x + 3y = -12 \end{matrix}$. Does the system have a solution? Explain.

15. What are the possible solutions for a linear system of two equations in two variables?

There are several ways to solve a linear system. The ***elimination-by-addition method*** is often used when we can easily eliminate one of the variables in the system by adding the equations.

Example 2: Solve the system $\begin{matrix} 3x + y = 9 \\ x - 2y = 10 \end{matrix}$ using the elimination-by-addition method.

In this system we can easily eliminate the x-terms after multiplying the second equation by –3, or we can eliminate the y-terms after multiplying the first equation by 2. We choose to eliminate the y-terms:

$$\begin{matrix} 3x + y = 9 \\ x - 2y = 10 \end{matrix} \quad ? \quad \begin{matrix} 6x + 2y = 18 \\ x - 2y = 10 \end{matrix}$$

Now we add the equations in the last system to eliminate the y-terms and solve the resulting equation for x:

$$\begin{array}{r} 6x + 2y = 18 \\ x - 2y = 10 \\ \hline 7x \quad\quad = 28 \end{array}$$

$$x = 4$$

This is the x-coordinate of the solution. To find the y-coordinate of the solution, we can substitute $x = 7$ into either of the *original* equations of the system and solve for y. We choose the first equation:

$$\begin{aligned} 3x + y &= 9 \\ 3(4) + y &= 9 \\ 12 + y &= 9 \\ y &= -3 \end{aligned}$$

The solution of the linear system is (4, –3). You should check this solution.

16. We will solve the linear system $\begin{matrix} 3x + y = 9 \\ x - 2y = 10 \end{matrix}$ of Example 1 by the ***substitution method***. The substitution method is often used when one of the equations can be easily solved for one of the variables without introducing fractions.
    a. Solve the first equation in the system for y and substitute the result into the second equation. (You could also solve the second equation for x and substitute the result into the first equation.)
    b. The second equation now contains only the variable x. Solve the equation for x.
    c. Determine the y-coordinate of the solution of the system.

For exercises 17-22, solve the linear system.

17. $\begin{matrix} 4x - y = 6 \\ x + y = 4 \end{matrix}$

18. $\begin{matrix} 2x - 5y = 9 \\ x - 4y = 3 \end{matrix}$

19. $\begin{matrix} x + 2y = 3 \\ 4x + 8y = 7 \end{matrix}$

20. $\begin{matrix} 5x + 7y = 1 \\ 4x - 2y = 16 \end{matrix}$

21. $\begin{matrix} 2x - 3y = -6 \\ -8x + 12y = 24 \end{matrix}$

22. $\begin{matrix} 0.2x + 0.3y = 1.7 \\ 0.5x + 0.7y = 2.9 \end{matrix}$

For exercises 23-26, write a linear system and solve the word problem. Write your answer in a complete sentence.

23. The total revenue from 245 tickets sold at a high school theater production was $1045. The price of a student ticket was $3 and the price of an adult ticket was $5. How many student tickets were sold? How many adult tickets?

24. Jeff has two student loans totaling $9600. One loan was made at an interest rate of 5% per year and the other was made at 8% per year. After one year, Jeff's loans accumulated $633 in interest. What was the original amount of each loan?

25. Ellen's car gets 18 miles per gallon (mpg) in the city, and 24 mpg on the highway. If she drives a total of 465 miles on 23 gallons of gas, how many miles were driven in the city, and how many miles were driven on the highway?

26. Tracey makes a $9.25 purchase at the local bookstore with a $20 bill. The store has no bills and gives her change in quarters and fifty-cent pieces. There are 30 coins in all. How many of each kind are there?