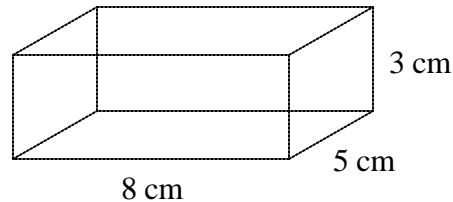


TOPIC 15: Surface Area and Volume

The **surface area** of a solid (three-dimensional) figure is the total area of its exterior surfaces. Surface area is measured in square units. For example, the box shown below has an exterior consisting of six surfaces. There are two 8 cm by 5 cm rectangles



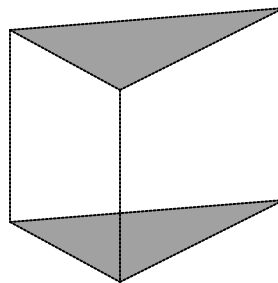
(on the bottom and top of the box), two 8 cm by 3 cm rectangles (on the front and back), and two 5 cm by 3 cm rectangles (on the right and left sides). We obtain the surface area S of the box by adding the areas of the rectangles:

$$\begin{aligned} S &= 2(8 \cdot 5) + 2(8 \cdot 3) + 2(5 \cdot 3) \\ S &= 80 + 48 + 30 \\ S &= 158 \text{ cm}^2 \end{aligned}$$

The **volume** of a solid figure is a measure of the amount of space occupied by the figure. Volume is measured in cubic units, such as cubic inches (in^3) or cubic centimeters (cm^3). We can think of volume as the number of unit cubes that fit *inside* the figure.

Now imagine filling up the box with unit cubes (cubes with each edge 1 cm). There will be three layers of cubes because the height of the box is 3 cm. The bottom of the box is an 8 cm by 5 cm rectangle, so there will be $8 \cdot 5 = 40$ cubes on the bottom layer. Of course, the other two layers will also have 40 cubes. Therefore the volume V of the box is given by $V = 8 \cdot 5 \cdot 3 = 120 \text{ cm}^3$. In general, the formula for the volume V of a box of length l , width w , and height h is $V = lwh$.

A special type of solid figure is a prism. A **prism** has two parallel bases that are congruent polygons (polygons that have the same size and shape). The sides of a prism are its **faces**. For example, a *triangular prism* is shown below:



Notice that the shaded triangular bases are in parallel planes and that there are three rectangular faces.

The box shown above is a rectangular prism whose bases are the rectangles on the top and bottom. The area of a base is $8 \cdot 5 = 40 \text{ cm}^2$, and multiplying this by the height gives the volume:

$$V = (\text{base area}) \cdot (\text{height})$$

$$V = (8 \cdot 5) \cdot 3$$

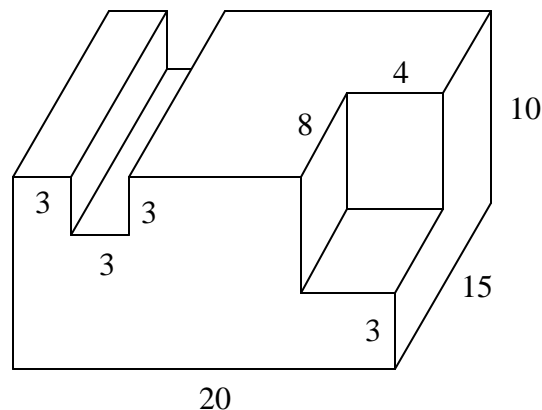
$$V = 120 \text{ cm}^3$$

In general, the formula for the volume V of a prism with base area B and height h is $V = Bh$.

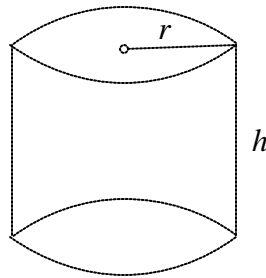
1. A **cube** is a rectangular prism, all of whose edges are the same length.
 - a. What is the formula for the surface area S of a cube with edges of length s ? Why?
 - b. What is the formula for the volume V of a cube with edges of length s ? Why?

For exercises 2-5, sketch the prism and find its surface area and volume.

2. A triangular prism with height 9 in whose bases are right triangles with sides 6 in, 8 in, and 10 in.
3. A square prism whose bases are squares with sides 4 cm and whose height is 7 cm.
4. A trapezoidal prism with height 12 ft. The bases are isosceles trapezoids (non-parallel sides congruent). The isosceles trapezoids have height 6 ft and bases 3 ft and 5 ft.
5. A hexagonal prism whose bases are regular hexagons with sides 2 m and whose height is 10 m.
6. A box is 20 in by 7 in by 5 in. How much wrapping paper is needed to wrap the box?
7. The figure shown below is a rectangular prism with two sections removed. Find its surface area and volume. The measurements are in centimeters.



A *cylinder* of radius r and height h is shown below:

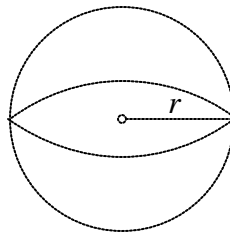


Notice that the top and bottom of a cylinder are circles. The formulas for the surface area S and volume V of a cylinder are:

$$S = 2\mathbf{pr}^2 + 2\mathbf{prh}$$

$$V = \mathbf{pr}^2h$$

A *sphere* of radius r , shown below, is the three-dimensional equivalent of a circle:



The formulas for the surface area S and the volume V of a sphere are:

$$S = 4\mathbf{pr}^2$$

$$V = \frac{4}{3}\mathbf{pr}^3$$

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8. The formula for the surface area S of a cylinder of radius r and height h is $S = 2\mathbf{pr}^2 + 2\mathbf{prh}$. What area does $2\mathbf{pr}^2$ represent? What area does $2\mathbf{prh}$ represent?
 9. The formula for the volume V of a cylinder of radius r and height h is $V = \mathbf{pr}^2h$. How is this formula similar to the formula for the volume of a prism?
 10. A soda can has a diameter of 5 cm and a height of 11.5 cm. Find the surface area and volume of the can.
 11. A ball has a diameter of 12 in. Find the surface area and volume of the ball.

12. Find the surface area and volume of the domed cylinder shown below. The measurements are in meters.

