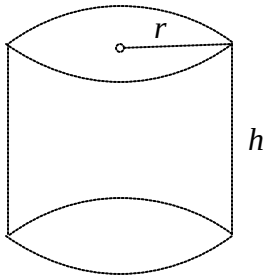
TOPIC 16: Soda Can Geometry

A company makes soda cans with a volume of $400\ \text{cm}^3$. The cost of a can depends on the amount of aluminum needed to make it, and the amount of aluminum needed is the surface area of the can. The company wants to minimize cost, so it needs to determine the size of the can that will minimize surface area.

We will assume the can is a cylinder with radius *r* and height *h*. The company must find values of *r* and *h* that produce a can of volume $400\ \text{cm}^3$ with minimum surface area. Recall the formulas for the surface area *S* and volume *V* of a cylinder:



$$S = 2\pi r^2 + 2\pi rh \qquad V = \pi r^2 h$$

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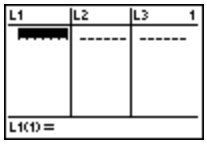
1. The volume of the can must be $400\ \text{cm}^3$, so $400 = \pi r^2 h$. Solve this formula for *h*.

2. The company will try different values of *r* and *h* to determine the minimum surface area. Copy and complete the table below in your notebook. Use $\pi = 3.14$, round height to the nearest tenth, and round surface area to the nearest whole number. The results for *r* = 1 are given.

r (cm)	h (cm)	$S = 2\pi r^2 + 2\pi rh \quad (\text{cm}^2)$
1	$\frac{400}{\pi(1^2)} \approx 127.4$	$S \approx 2\pi(1^2) + 2\pi(1)(127.4)$ $S \approx 806$
2		
3		
4		
5		
6		
7		
8		

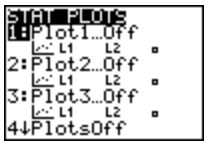
It appears from the table above that the minimum surface area occurs when $r = 4$ cm and $h \approx 8.0$ cm. However, there may be other values of r and h that produce a smaller surface area. To explore this possibility we will graph the information in the table.

We start by entering the data into *lists.* Press the STAT key, then press ENTER. Your screen should look like the one shown below:



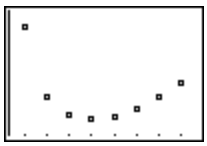
Enter the values of r into list L1, the values of h into L2, and the values of S into L3.

The surface area (and height) of the can depends on the radius, so we will make a graph with r on the x-axis and S on the y-axis. Start by pressing the 2nd key, then Y= ; the following STATPLOT menu should appear on the screen:



To edit 1:Plot1, press ENTER. Use the cursor and the ENTER key to highlight each option as shown below. Press the 2nd key, then 3 to select L3 for Ylist.

Now press the ZOOM key. This will allow us to automatically create a viewing window to see the graph. Scroll down to 7:ZoomStat and press ENTER. You should see the graph shown below:



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3. Use the results of exercise 1 above to write a formula for the surface area of the can in terms of r only.

4. Use your calculator to graph the formula in exercise 3 above. Press the Y= key and type in the formula, using the variable x in place of r. Now press GRAPH. Does the curve go through the points on the graph shown above?

5. Use the TRACE feature of your calculator to explore points on the curve. What value of r gives the minimum surface area? What is the corresponding value of h? What is the minimum surface area? Compare your answers with other students.